APPENDIX: Calculating risk of hip fracture according to age and bone mineral density

To partition the total number of hip fractures into those experienced by women with and without osteoporosis based on each definition, we set up the following simultaneous equations for each age group (i.e., 50-54, 55-59, 60-64.....80-84, 85+) and solved for the risk of fracture given osteoporosis (defined here by the T-score):

$$\text{(I)}\quad \text{Total hip fractures} = \text{hip fractures} \mid \text{osteoporosis} + \text{hip fractures} \mid \sim \text{osteoporosis}$$

$$= N_{osteop} * \text{fxrisk} \mid \text{osteoporosis} + N_{\sim osteop} * \text{fxrisk} \mid \sim \text{osteoporosis}$$

$$\text{(II)}\quad \frac{\text{fxrisk} \mid \text{osteop}}{\text{fxrisk} \mid \sim \text{osteop}} = RR_{\text{osteop vs. } \sim \text{osteop}}$$

Rearranging equation (II) shows that:

$$\text{fxrisk} \mid \text{osteop} = RR * \text{fxrisk} \mid \sim \text{osteop}$$

By substitution, then:

$$\text{Total hip fractures} = N_{osteop} * RR * \text{fxrisk} \mid \sim \text{osteop} + N_{\sim osteop} * \text{fxrisk} \mid \sim \text{osteop};$$

$$= \text{fxrisk} \mid \sim \text{osteop} * (N_{osteop} * RR + N_{\sim osteop});$$

Therefore,

$$\boxed{\frac{\text{Total hip fractures}}{(N_{osteop} * RR + N_{\sim osteop})} = \text{fxrisk} \mid \sim \text{osteop}}$$

All terms on the left side of this equation are known (total hip fractures from NIS; N_{osteop} and $N_{\sim osteop}$ from NHANES) except for the RR. This term was calculated based on the work of Cummings, et. al. (Cummings SR, Bates D, Black DM. Clinical use of bone densitometry. JAMA 2002;288:1889-1897, Table 3) which provides the lifetime risk of hip fracture at various T-score thresholds. We calculated the average T-score for women for each of the 3 osteoporosis definitions and for women without osteoporosis. For example, for ages 50-54 years, the average T-score for women under

the osteopenic definition (i.e., in our equation osteoporosis would mean T-score<-1.5, and ~osteoporosis would meant T-score≥-1.5) was -2.1. For women in the no osteoporosis group, the average T-score was -0.40. Using linear interpolation the lifetime risk of hip fracture for these women is 28.2% and 12.4%, respectively, corresponding to an RR = 28.2/12.4 = 2.27. Substituting RR=2.27 into the equation yields the following:

$$\text{fxrisk} \mid \sim \text{osteop} = \frac{44{,}157}{(2{,}123{,}255 * 2.27 + 6{,}854{,}569)} = 0.00378$$

that is, the 10-year chance of fracture for women in the no osteoporosis group is about 0.378%. Since RR=2.27, the corresponding chance of hip fracture for women in the osteoporosis group is:

2.27*0.378% or 0.86%

The corresponding 10-year hip fracture risk for women with osteoporosis -- based on the proposed definition (T<-2.0) -- is calculated as follows:

$$\text{fxrisk} \mid \text{osteop} = \text{fxrisk} \mid \sim\text{osteop} * \text{RR}$$
$$= 0.00378 * 2.6 = 0.97\%$$

again deriving the RR based on the Cummings table using the average T-scores for women in the proposed definition and the no osteoporosis groups.

Finally, the fracture risk for the current definition (T-score < -2.5) is determined in the same way:

$$\text{fxrisk} \mid \text{osteop} = \text{fxrisk} \mid \sim\text{osteop} * \text{RR}$$
$$= 0.00378 * 3.2 = 1.2\%$$